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ON A METHOD OF STUDYING STEADY-STATE OSCILLATIONS OF AN ELASTIC HALF-SPACE CONTAINING A CAVITY

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A method is proposed by means of which it is possible to study a plane or axisymmetric boundary-value problem of the theory of elasticity for the steady-state oscillations of an elastic half-space containing a circular, cylindrical or spherical cavity. A superposition principle is applied, allowing the problem to be reduced to the solution of a system of integral equations of the second kind. Features of unknown stress functions in the neighborhood of the corner points are studied. After these features have been determined, the solution of the problem is reduced to the solution of a quasi-complete regular infinite system of linear algebraic equations. By means of this method, it is possible to study the case in which the cavity reaches the surface of the medium.

1. Let us consider the plane problem of determining the steady-state oscillations of an elastic half-space with a cavity in the form of a circular cylinder whose axis is parallel to the surface of the half-space.

The motion of the points of the medium is described by dynamic displacement equations of the theory of elasticity, called Lamé equations /1/. The region filled by the elastic medium is given, in a Cartesian rectangular coordinate system thus: $x \geqslant 0$, $r \geqslant a$, (where a is the radius of the hole, and $x = h$, $y = 0$ are the coordinates of its center). The following boundary conditions are imposed on the boundary of the region:

$$x = 0,\ \ \sigma_x = t_1(y)\, e^{-i\omega t},\ \tau_{xy} = t_2(y)\, e^{-i\omega t} \tag{1.1}$$
$$r = a,\ \ \sigma_r = \tau_1(\varphi)\, e^{-i\omega t},\ \tau_{r\varphi} = \tau_2(\varphi)\, e^{-i\omega t}$$
$$\left(r = \sqrt{(x-h)^2 + y^2},\quad \varphi = \operatorname{arctg} \frac{y}{x-h}\right)$$

Here (r, φ) is the local cylindrical coordinate system bound with the cylindrical hole and ω is the frequency of the steady-state oscillations.

At infinity the components of the displacement and stress vectors are decreasing and tend to zero.

The solution of our boundary-value problem may be constructed in the form of a sum of the solutions of the following two boundary-value problems: the problem of determining the steady-state oscillations of a homogeneous elastic half-space affected by stresses

$$x = 0,\ \sigma_x = X_1(y)\, e^{-i\omega t},\ \tau_{xy} = X_2(y)\, e^{-i\omega t}$$

acting on the surface, and the problem of determining the steady-state oscillations of an infinite elastic space with a cylindrical notch at the surface of which a load

$$r = a,\ \sigma_r = Y_1(\varphi)\, e^{-i\omega t},\ \tau_{xy} = Y_2(\varphi)\, e^{-i\omega t}$$

is applied.

By combining the solutions of the above two problems, we may satisfy the boundary conditions (1.1) of the initial problem. To determine the functions $X_j(y)$ and $Y_j(\varphi)$ $(j = 1, 2)$, we obtain a system of four integral equations:

$$X_k(y) + \frac{1}{2\pi} \int_{\gamma_0}^{2\pi-\gamma_0} Y_1(\eta) \sum_{n=-\infty}^{\infty} \Phi_{k1}^{(n)}(y)\, e^{-in\eta}\, d\eta + \tag{1.2}$$

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$$\frac{1}{2\pi}\int_{\gamma_0}^{2\pi-\gamma_0} Y_2(\eta)\sum_{n=-\infty}^{\infty}\Phi_{k2}^{(n)}(y)e^{-in\eta}\,d\eta = t_k(y)$$

$$Y_k(\varphi)+\int_{-\infty}^{\infty}X_1(\eta)\int_\sigma \Psi_{k1}(u,\varphi)e^{iu(\eta-y_0)}\,d\eta\,du +$$

$$\int_{-\infty}^{\infty}X_2(\eta)\int_\sigma \Psi_{k2}(u,\varphi)e^{iu(\eta-y_0)}\,d\eta\,du = \tau_k(\varphi),\quad k=1,2$$

$$\gamma_0=\begin{cases}\arccos\frac{h}{a}, & |h|<a\\ 0, & |h|\geqslant a\end{cases}$$

Whenever the cavity reaches the surface of the medium ($|h|<a$), the required functions $Y_k(\varphi)$ $(k=1,2)$ in the region $\varphi<\gamma_0$, $\varphi>2\pi-\gamma_0$ are assumed equal to zero. The choice of the contour σ in (1.2) is dictated by the principle of limiting absorption /2/. In our case, the contour encompasses the negative pole and the branching point of the functions $\Psi_{kj}(u,\varphi)$ from above, and the positive poles underneath, and otherwise coincides with the real axis. The right half of the contour that lies in the half-plane $\operatorname{Re} u\geqslant 0$ is denoted henceforth by σ^+.

We present expressions for $\Phi_{11}^{(n)}(y)$ and $\Psi_{11}(u,\varphi)$:

$$\Phi_{11}^{(n)}(y) = [a_{2n}E_{1n} - a_{1n}E_{2n}]\,[b_{1n}a_{2n} - b_{2n}a_{1n}]^{-1}$$

$$E_{1n}(y) = 2\mu\Big\{\Big[\frac{n(n-1)}{r_0^2}(\cos 2\varphi_0 - i\sin 2\varphi_0) - \Big(\frac{\lambda}{\mu}+1+\cos 2\varphi_0\Big)\frac{\theta_2^2}{2}\Big]H_n^{(1)}(\theta_1 r_0) + \frac{\theta_1}{r_0}(\cos 2\varphi_0 - i\sin 2\varphi_0)H_{n+1}^{(1)}(\theta_1 r_0)\Big\}$$

$$E_{2n}(y) = 2i\mu\Big\{\Big[\frac{n(n-1)}{r_0^2}(\cos 2\varphi_0 - i\sin 2\varphi_0) + i\frac{\theta_2^2}{2}\Big]H_n^{(1)}(\theta_2 r_0) - \frac{\theta_2}{r_0}(n\cos 2\varphi_0 + i\sin 2\varphi_0)H_{n+1}^{(1)}(\theta_2 r_0)\Big\}$$

$$a_{1n} = \frac{2i\mu n}{a}\Big[\frac{n-1}{a}H_n^{(1)}(\theta_1 a) - \theta_1 H_{n+1}^{(1)}(\theta_1 a)\Big]$$

$$a_{2n} = -2\mu\Big\{\Big[\frac{n(n-1)}{a^2} - \frac{\theta_2^2}{2}\Big]H_n^{(1)}(\theta_2 a) + \frac{\theta_2}{a}H_{n+1}^{(1)}(\theta_2 a)\Big\}$$

$$b_{1n} = 2\mu\Big\{\Big[\frac{n(n-1)}{a^2} - \Big(\frac{\lambda}{2\mu}+1\Big)\theta_1^2\Big]H_n^{(1)}(\theta_1 a) + \frac{\theta_1}{a}H_{n+1}^{(1)}(\theta_1 a)\Big\}$$

$$b_{2n} = \frac{2i\mu n}{a}\Big[\frac{n-1}{a}H_n^{(1)}(\theta_2 a) - \theta_2 H_{n+1}^{(1)}(\theta_2 a)\Big]$$

$$r_0 = r|_{x=0} = \sqrt{h^2+y^2};\quad \varphi_0 = \operatorname{arctg}(-y/h)$$

$$\Psi_{11}(u,\varphi) = \frac{1}{2\pi\delta}\Big\{(2u^2-\theta_2^2)\Big[\frac{\lambda}{\mu}\Big(\frac{\sigma_1}{a}\cos\varphi + i\frac{u}{a}\sin\varphi - \theta_1^2\Big) + 2(u^2\cos 2\varphi - \theta_1^2\cos^2\varphi) + 2i\sigma_1 u\sin 2\varphi\Big]e^{-\sigma_1 x_0} - 2\sigma_1 u\Big[\frac{\lambda}{\mu}\Big(\frac{u}{a}\cos\varphi + i\frac{\sigma_2}{a}\sin\varphi\Big) + 2\sigma_2 u\cos 2\varphi + i(2u^2-\theta_2^2)\sin 2\varphi\Big]e^{-\sigma_2 x_0}\Big\}$$

$$\sigma_j^2 = u^2-\theta_j^2;\quad \theta_1^2 = \frac{\rho\omega^2}{\lambda+2\mu},\quad \theta_2^2 = \frac{\rho\omega^2}{\mu},\quad j=1,2$$

$$\delta = (\sigma_2^2+u^2)^2 - 4\sigma_1\sigma_2 u^2,\ x_0 = x|_{r=a} = h - a\cos\varphi$$

$$y_0 = y|_{r=a} = -a\sin\varphi$$

Here ($H_n^{(1)}(z)$ are Hankel functions of the first kind, ρ is the density of the medium, and λ and μ are the Lamé coefficients. Because of the rest functions are cumbersome, they are not presented here.

The system (1.2) may be considered in a space of integrable functions. With this class of functions, the energy determined by a solution of the dynamic problems of the theory of elasticity and bounded within any finite volume will be finite.

Note that the operators in the right side of (1.2) are not continuous in a space of integrable functions whenever the cavity reaches the surface of the half-space ($|h|<a$). Therefore, in these cases it is necessary to construct a system regularizer. This construction

reduces to the identification of the singular part of the kernels of the system /2/. To identify a bounded operator that is not completely continuous, we studied the behavior of the functions $\Phi_{ij}^{(n)}(y)$ as $n \to \infty$ and $\Psi_{kj}(u, \varphi)$ as $|u| \to \infty$.

We now investigate the behavior of the required functions in a neighborhood of the corner points of the region. For this purpose, we substitute the limiting values of the functions $\Phi_{ij}^{(n)}$ and Ψ_{kj} in the system (1.2) and undertake the substitution of variables $\zeta = y - \sqrt{a^2 - h^2}$, $\chi = \varphi - \gamma_0$. Applying a Mellin /3/ transformation with respect to the parameters ζ and χ to the resulting system (the functions $Y_k(\varphi)$, $\Psi_{kj}^{(n)}(\varphi)$, and $\tau_k(\varphi)$ are continued analytically for this purpose to the region $\varphi > 2\pi - \gamma_0$), we obtain a system of linear algebraic equations in the Mellin transform of the unknown functions:

$$\begin{aligned}
&\mathbf{AX} = \mathbf{B},\ \mathbf{A} = \| a_{ij} \| \ (i, j = 1, 2, 3, 4) \\
&\mathbf{X} = \operatorname{col}(Y_1(p), Y_2(p), X_1(p), X_2(p)) \\
&\mathbf{B} = \operatorname{col}(t_1(p), t_2(p), \tau_1(p), \tau_2(p)) \\
&a_{11} = \frac{1}{\sin p\pi} p \sin\gamma_0 \cos[p(\pi - \gamma_0) + \gamma_0], \quad a_{22} = -a_{11} \\
&a_{12} = \frac{p}{\sin p\pi} \sin\gamma_0 \sin[p(\pi - \gamma_0) + \gamma_0], \quad a_{21} = a_{12} \\
&a_{33} = \frac{\sin(\pi - \gamma_0)}{\sin p\pi}, \quad a_{34} = \frac{2\sin\gamma_0}{\sin p\pi} \sin[p(\pi - \gamma_0) + \gamma_0] \\
&a_{44} = \frac{\sin[p(\pi - \gamma_0) + 2\gamma_0]}{\sin p\pi}; \quad a_{13} = a_{24} = a_{31} = a_{42} = 1 \\
&a_{14} = a_{23} = a_{32} = a_{41} = a_{43} = 0; \ \gamma_0 = \arccos\frac{h}{a}
\end{aligned} \tag{1.3}$$

Here p is the parameter of the Mellin transformation, and γ_0 is the angle in the plane $z = 0$ between the y-axis and the tangent to the circle at the corner point.

Solving the system (1.3) by Cramer's rule /4/, we obtain the expressions

$$\begin{aligned}
&Y_k(p) = \frac{\Delta_k(p)}{\Delta(p)}, \quad X_k(p) = \frac{\Delta_{k+2}(p)}{\Delta(p)} \quad (k = 1, 2) \\
&\Delta(p) = 1 + 2p\frac{\sin^2\gamma_0}{\sin^2\pi p}\cos[2p(\pi - \gamma_0) + 2\gamma_0] - \\
&\quad \frac{p^2\sin^2\gamma_0}{\sin^4 p\pi}\sin[(\pi - \gamma_0)p + 2\gamma_0]\sin p(\pi - \gamma_0)
\end{aligned} \tag{1.4}$$

for the Mellin transform of the required functions.

To convert formulas (1.4), it is necessary to determine the roots of the equation $\Delta(p) = 0$, bearing in mind that $\operatorname{Re} p \in (0, 1)$. The equation $\Delta(p) = 0$ was studied numerically on a computer, and it was established that with fixed γ_0, it has only a single root p_0; the relation between p_0 and γ_0 is presented in the Fig.1. Thus the residue theorem may now be applied.

Applying the inverse Mellin transformation to (1.4), we find that in a neighborhood of a singular point,

$$\begin{aligned}
&Y_k(\varphi) \sim \frac{C_k}{(\varphi - \gamma_0)^{p_0}}, \quad X_k(y) \sim \frac{C_{k+2}}{(y - y_0)^{p_0}} \quad (k = 1, 2) \\
&c_j = \Delta_j(p_0)/\Delta'(p_0),\ j = 1, 2, 3, 4,\ y_0 = \sqrt{a^2 - h^2}
\end{aligned}$$

The system of integral equations (1.2) may be reduced to an infinite system of linear albebraic equations of the form

$$\begin{aligned}
&Y_j^{(k)} - \sum_{n=-\infty}^{\infty} F_{j1k}^{(n)} Y_1^{(n)} - \sum_{n=-\infty}^{\infty} F_{j2k}^{(n)} Y_2^{(n)} = \tau_j^{(k)} - T_j^{(k)}; \\
&j = 1, 2; \quad k = 0, 1, 2 \ldots
\end{aligned} \tag{1.5}$$

Here

$$\begin{aligned}
&F_{\alpha\beta n}^{(k)} = \frac{1}{2\pi} \int_\sigma \int_{-\infty}^{\infty} \int_0^{2\pi} [\Phi_{1\beta}^{(n)}(y)\Psi_{\alpha 1}(u, \varphi) + \Phi_{2\beta}^{(n)}(y)\Psi_{\alpha 2}(u, \varphi)] \times \\
&\quad \exp[iu(y - y_0) - ik\varphi]\, d\varphi\, dy\, du; \quad \alpha = 1, 2; \quad \beta = 1, 2 \\
&T_\alpha^{(k)} = \frac{1}{2\pi} \int_\sigma \int_{-\infty}^{\infty} \int_0^{2\pi} [t_1(y)\Psi_{\alpha 1}(u, \varphi) + t_2(y)\Psi_{\alpha 2}(u, \varphi)] \times \\
&\quad \exp[iu(y - y_0) - ik\varphi]\, d\varphi\, dy\, du, \quad \alpha = 1, 2
\end{aligned}$$

$$Y_j^{(k)}=\frac{1}{2\pi}\int_0^{2\pi} Y_j(\varphi)\,e^{-ik\varphi}\,d\varphi,\quad \tau_j^{(k)}=\frac{1}{2\pi}\int_0^{2\pi}\tau_j(\varphi)\,e^{-ik\varphi}\,d\varphi,\quad j=1,2$$

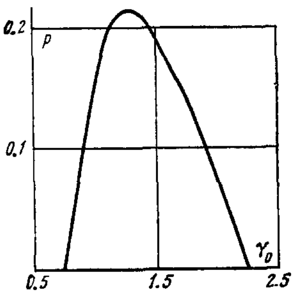

Fig.1

The system (1.5) is quasi-complete by regular if $|h|>a$, since it approximates a system of integral equations with completely continuous operator. In the case $|h|<a$, we arrive at a system of the form (1.5) after regularization of the system of integral equations (1.2). In the new system, only the form of the integrand functions (which determine the coefficients of the system) changes. In this case, the system of linear equations (1.5) will also be quasi-completely regular, understood as a system generated by a completely continuous operator.

The solution of the system (1.5) in both cases may be effectively carried out numerically on a computer by using the truncation method and estimating the virtual convergence of the process.

2. Let us consider the problem of determining the steady-state oscillations of an elastic half-space containing a spherical cavity in an axisymmetric formulation.

The motion of the points of the medium is described by the Lamé dynamic equations. In a cylindrical coordinate system (R, z), the medium occupies the region $z\leqslant 0$, $r=\sqrt{R^2+(z+h)^2}\geqslant a$ where a is the radius of the spherical cavity, and $R=0$ and $z=-h$ are the coordinates of its center. In the case of axial symmetry, the cylindrical coordinates (R, z) are related to the spherical coordinates (r, α) by the relations

$$r=\sqrt{R^2+(z+h)^2},\quad \alpha=\operatorname{arctg}\frac{R}{h+z},\quad R=r\sin\alpha$$
$$z=r\cos\alpha-h$$

Suppose a load

$$z=0,\ \sigma_z=t_1(R)\,e^{-i\omega t},\ \tau_{Rz}=t_2(R)\,e^{-i\omega t}$$
$$r=a,\ \sigma_r=\tau_1(\alpha)\,e^{-i\omega t},\ \tau_{r\alpha}=\tau_2(\alpha)\,e^{-i\omega t} \tag{2.1}$$

oscillating with frequency ω is applied at the boundary of the region.

At infinity the components of the displacement and stress vectors are decreasing and tend to zero.

The solution of this boundary-value problem is constructed as in the case of a cylindrical hole using the principle of super-position in the form of a sum of the solutions of the boundary-value problems for determining the steady-state axisymmetric oscillations of an elastic half-space affected by the harmonic loads

$$z=0,\ \sigma_z=X_1(R)\,e^{-i\omega t},\ \tau_{Rz}=X_2(R)\,e^{-i\omega t}$$

and the problem of determining the vibrations of an infinite elastic space with spherical cavity of radius a at whose surface are specified the stresses

$$r=a,\ \sigma_r=Y_1(\alpha)\,e^{-i\omega t},\ \tau_{r\alpha}=Y_2(\alpha)\,e^{-i\omega t}$$

If the boundary conditions (2.2) of the initial boundary-value problem are met, as in Sect.1, we arrive at the following system of integral equations of the second kind for determining the unknown functions $X_j(R)$ and $Y_j(\alpha)$:

$$X_k(R)+\int_{\gamma_0}^{\pi} Y_1(\psi)\sin\psi\sum_{n=0}^{\infty}\Phi_{k1}^{(n)}(R)\,P_n(\cos\psi)\,d\psi+ \tag{2.2}$$
$$\int_{\gamma_0}^{\pi} Y_2(\psi)\sin\psi\sum_{n=0}^{\infty}\Phi_{k2}^{(n)}(R)\frac{\partial P_n(\cos\psi)}{\partial\psi}\,d\psi=t_k(R)$$
$$Y_k(\alpha)+\int_{\rho_0}^{\infty}\beta X_1(\beta)\int_{\sigma^+}\Psi_{k1}(u,\alpha)\,J_1(\beta u)\,u\,du\,d\beta+$$
$$\int_{\rho_0}^{\infty}\beta X_2(\beta)\int_{\sigma^+}\Psi_{k2}(u,\alpha)\,J_0(\beta u)\,u\,du\,d\beta=\tau_k(\alpha),\quad k=1,2$$
$$\gamma_0=\begin{cases}\arccos(h/a), & |h|<a,\\ 0, & |h|\geqslant a,\end{cases}\qquad \rho_0=\begin{cases}\sqrt{a^2-h^2}, & |h|<a,\\ 0, & |h|\geqslant a\end{cases}$$

Here $P_n(\cos\psi)$ are Legendre polynomials and $J_n(x)$ are Bessel functions. The choice of the contour σ^+ is directed by the principle of limiting absorption.

Since the functions $\Phi_{kj}^{(n)}(R)$ and $\Psi_{kj}(u, \alpha)$ have a cumbersome form, we present only one example of each functions:

$$\Phi_{11}^{(n)} = \frac{2n+1}{2} \frac{b_{2n}F_{1n}(R) - b_{1n}F_{2n}(R)}{a_{1n}b_{2n} - a_{2n}b_{1n}}$$

$$F_{1n}(R) = \frac{2\mu}{\theta_1^2\sqrt{r_0}}\left\{\frac{1}{r_0}\left[\frac{h^2n(n-1) - R^2n}{r_0^4} - h^2\theta_1^2\right] \times\right.$$
$$H_{n+1/2}^{(1)}(\theta_1 r_0)P_n(\cos\alpha) - \frac{2n-1}{r_0^4} RhH_{n+1/2}^{(1)}(\theta_1 r_0)\frac{\partial P_n(\cos\alpha)}{\partial\alpha} +$$
$$\frac{\theta_1(2h^2 - R^2)}{r_0^3} H_{n+3/2}^{(1)}(\theta_1 r_0)P_n(\cos\alpha) + \frac{2Rh\theta_1}{r_0^3} H_{n+3/2}^{(1)}(\theta_1 r_0)\times$$
$$\left.\frac{\partial P_n(\cos\alpha)}{\partial\alpha} - \frac{1}{2\mu}\theta_1^2 H_{n+1/2}^{(1)}(\theta_1 r_0)P_n(\cos\alpha)\right\}_{\alpha=\alpha_0}$$

$$F_{2n}(R) = \frac{2\mu}{\theta_2^2\sqrt{r_0}}\left\{\frac{1}{r_0^4}[h^2(n-1) - R^2n]H_{n+1/2}^{(1)}(\theta_2 r_0)P_n(\cos\alpha) -\right.$$
$$\frac{Rh}{r_0^4}\left[\frac{2n-1}{nr_0^2} - \frac{\theta_2^2}{n(n+1)}\right]H_{n+1/2}^{(1)}(\theta_2 r_0)\frac{\partial P_n(\cos\alpha)}{\partial\alpha} + \frac{\theta_2(R^2 - h^2)}{r_0^3}\times$$
$$\left.H_{n+3/2}^{(1)}(\theta_2 r_0)P_n(\cos\alpha) - \frac{Rh\theta_2}{r_0^3 n(n+1)} H_{n+3/2}^{(1)}(\theta_2 r_0)\frac{\partial P_n(\cos\alpha)}{\partial\alpha}\right\}_{\alpha=\alpha_0}$$

$$a_{1n} = \frac{1}{\sqrt{a}}\left\{\left[2\mu\frac{n(n-1)}{\theta_1^2a^2} - \lambda - 2\mu\right]H_{n+1/2}^{(1)}(\theta_1 a) + \frac{4\mu}{\theta_1 a}H_{n+3/2}^{(1)}(\theta_1 a)\right\}$$
$$a_{2n} = \frac{2\mu}{\theta_2^2a\sqrt{a}}\left[\frac{n-1}{a}H_{n+1/2}^{(1)}(\theta_2 a) - \theta_2 H_{n+3/2}^{(1)}(\theta_2 a)\right]$$
$$b_{1n} = \frac{2\mu}{\theta_1^2a\sqrt{a}}\left[\frac{n-1}{a}H_{n+1/2}^{(1)}(\theta_1 a) - \theta_1 H_{n+3/2}^{(1)}(\theta_1 a)\right]$$
$$b_{2n} = \frac{\mu}{\theta_2^2n(n+1)\sqrt{a}}\left\{\left[\frac{2(n^2-1)}{a^2} - \theta_2^2\right]H_{n+1/2}^{(1)}(\theta_2 a) +\right.$$
$$\left.\frac{2\theta_2}{a}H_{n+3/2}^{(1)}(\theta_2 a)\right\}$$
$$r_0 = r|_{z=0} = \sqrt{h^2 + R^2},\ \alpha_0 = \operatorname{arctg}(R/h)$$
$$\Psi_{12}(u, \alpha) = \frac{2u}{\delta}\{\sigma_2[(\sigma_2^2 + u^2)e^{\sigma_2 z_0} - 2u^2e^{\sigma_1 z_0}][uJ_0(R_0u) -$$
$$R_0^{-1}J_1(R_0u)]\sin^2\alpha - u\sigma_2[(\sigma_2^2 + u^2)e^{\sigma_2 z_0} - 2\sigma_1^2e^{\sigma_1 z_0}]\times$$
$$J_0(R_0u)\cos^2\alpha + [(\sigma_2^2 + u^2)e^{\sigma_2 z_0} - 4u^2\sigma_1\sigma_2e^{\sigma_1 z_0}]J_1(R_0u)\times$$
$$\sin\alpha\cos\alpha\} + \frac{2\lambda\sigma_2}{\mu\delta}(\sigma_1^2 - u^2)u^2e^{\sigma_1 z_0}J_0(R_0u);$$
$$z_0 = z|_{r=a} = a\cos\alpha - h,\ R_0 = R|_{r=a} = a\sin\alpha$$

The remaining parameters were described in Sect.1. Note that the basic properties of the elements of the system (2.2) are analogous to the properties of the system (1.2).

In the case $|h| < a$ (the spherical cavity crosses the surface of the half-space), in the foregoing it is necessary to regularize the system (2.2). For this purpose, we investigate the behavior of the unknown functions in a neighborhood of the corner points. As in the problem of Sect.1, the Mellin transforms $X_j(p)$ and $Y_j(p)$ of the required functions are determined by the relations (1.4), i.e., the unknown functions for the problem of determining the spherical and cylindrical holes in an elastic half-space have the same features at the corner points of the region. The system (2.2) may be reduced to an infinite quasi-complete regular system of linear algebraic equations of the form (1.5) ($|h| > a$) where

$$Y_\alpha^{(k)} = \int_0^\pi Y_\alpha(\varphi)\sin\varphi D_\alpha P_k(\cos\varphi)\,d\varphi,\quad \tau_\alpha^{(k)} = \int_0^\pi \tau_\alpha(\varphi)\sin\varphi\times \tag{2.3}$$
$$D_\alpha P_k(\cos\varphi)\,d\varphi,\quad D_\alpha = \begin{cases} 1, & \alpha = 1 \\ \partial/\partial\varphi, & \alpha = 2.\end{cases}$$
$$F_{\alpha\beta k}^{(n)} = \int_{\sigma^+}\int_0^\infty\int_0^\pi [\Phi_{1\beta}^{(n)}(R)\Psi_{\alpha 1}(u, \varphi)J_0(Ru) +$$
$$\Phi_{2\beta}^{(n)}(R)\Psi_{\alpha 2}(u, \varphi)J_1(Ru)]R\sin\varphi D_\alpha P_k(\cos\varphi)\,d\varphi\,dR\,du$$
$$T_\alpha^{(k)} = \int_{\sigma^+}\int_0^\infty\int_0^\pi [t_1(R)\Psi_{\alpha 1}(u, \varphi)J_0(Ru) + t_2(R)\Psi_{\alpha 2}(u, \varphi)\times$$
$$J_1(Ru)]R\sin\varphi D_\alpha P_k(\cos\varphi)\,d\varphi dRdu$$
$$(\alpha, \beta = 1,2;\ k, n = 0, 1, 2, \ldots)$$

In the case $|h| < a$, an analogous system is obtained after regularization of the system (2.2). Here only the outward form of the coefficients of $F_{\alpha\beta k}^{(n)}$ and $T_\alpha^{(k)}$ in (2.3) changes, while the basic properties are retained.

In conclusion, note that if this method is used to study steady-state oscillations in regions containing corner points, the order of the singularities of the unknown functions in a neighborhood of the corner point is determined by the geometry of the section in the coordinate plane containing the corner point. It is precisely in studying in an analogous fashion the problem for wedge-shaped regions obtained from the superposition of two half-spaces in a two-dimensional formulation that the order of the singularities will be somewhat different than in the case of the problem of determining a spherical or cylindrical cavity (for an angle of opening of a plane wedge equal to the angle γ_0 between the plane and tangent to the section of the sphere of cylinder at the corner point). From the latter fact, we may conclude that the presence of curvature in one of the lines in the coordinate section containing the corner point alters the order of the singularity of the unknown functions determined from the integral equation to which the solution of the boundary-value problem reduces in this approach.

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